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Fuzzy Hyper-Connected Spaces

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ABSTRACT

In this paper the concept of fuzzy hyper-connected spaces is discussed. Investigation on several characterizations of fuzzy Hyper-connected space and the relations of fuzzy Hyper-connected space and some fuzzy topological spaces are studied.

Keywords: Fuzzy dense, fuzzy nowhere dense, fuzzy first category, fuzzy second category, fuzzy Baire space, fuzzy D-Baire space and fuzzy hyper-connected space.

I. INTRODUCTION

The theory of fuzzy sets was initiated by L. A. Zadeh in his classical paper [8] in the year 1965 as an attempt to develop a mathematically precise framework to treat systems or phenomena which cannot themselves be characterized precisely. The potential of fuzzy notion was analised by the researchers and has successfully been applied for investigations in all the branches of Science and Technology. The paper of C. L. Chang [2] (1968) paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

The concepts of Hyper-connected spaces have been studied in classical topology [3]. In this paper we studied fuzzy Hyper-connected spaces and investigation on several characterizations of fuzzy Hyper-connected space and the relations of fuzzy Hyper-connected space and some fuzzy topological spaces.

II. PRELIMINARIES

Now reviews of some basic notions and results are used in the sequel. In this work by (X,T) or simply by X, we can denote a fuzzy topological space due to Chang [2].

Definition 2.1 [1]

Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T). Then we define:

$\lambda \vee \mu : X \rightarrow [0,1]$ as follows: $\lambda \vee \mu\ (x) = \max \{\lambda(x), \mu(x)\}$;

$\lambda \wedge \mu : X \rightarrow [0,1]$ as follows: $\lambda \wedge \mu\ (x) = \min \{ \lambda(x), \mu(x)\}$;

$\mu = \leftrightarrow \mu(x) = 1- \lambda(x)$.

For a family $\lambda_i \in I$ of fuzzy sets in (X, T), the union $\psi = U_i \lambda_i$ and intersection $\delta = {}_i\lambda_i$ are defined respectively as $\psi(x) = Sup_i\{\lambda_i(x), x \in X\}$, and $\delta(x) = Inf_i\{\lambda_i(x), x \in X\}$.

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Definition 2.2 [2]

Let (X,T) be a fuzzy topological space. For a fuzzy set λ of X, the interior and the closure of λ are defined respectively as $int(\lambda)=\{\mu\backslash\mu\leq\lambda,\ \mu\in T\}$ and $cl(\lambda)=\{\mu\backslash\lambda\leq\mu,\ 1-\mu\in T\}$.

Definition 2.3 [7]

A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy dense if there exists no fuzzy closed set μ ϵ (X,T) such that λ < μ < 1. That is cl(λ) = 1.

Definition 2.4 [6]

A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that μ < cl(λ). That is, int cl(λ) = 0.

Definition 2.5 [6]

Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called fuzzy first category set if $\lambda = \cup_{i=1}^{\infty} \lambda_i$, where λ_i's are fuzzy nowhere dense sets in (X,T). A fuzzy set which is not fuzzy first category set is called a fuzzy second category set in (X,T).

Definition 2.6 [5]

A fuzzy topological space (X,T) is called fuzzy first category if $1 = \cup_{i=1}^{\infty} \lambda_i$, where λ_i's are fuzzy nowhere dense sets in (X,T). A topological space which is not of fuzzy first category is said to be of fuzzy second category.

Definition 2.7 [6]

Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy Baire space if $int(\cup_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T).

Definition 2.8 [5]

Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy D-Baire space if each fuzzy first category set is fuzzy nowhere dense sets in (X,T).

Definition 2.9 [4]

A fuzzy topological space X is said to be fuzzy hyper connected space if every non null open subset of X is fuzzy dense in X.

III. FUZZY HYPER-CONNECTED SPACES

Motivated by the classical concept studies by [3] L.A Steen and J.A. Seebach, Jr., Counter examples in topology, Holt, Rinehart and Winston, Inc., U.S.A., 1970. We shall now define:

Definition: 3.1

A fuzzy topological space (X, T) is said to be a hyper-connected space in which every non-empty open set is dense in (X,T).

Example 3.1

Let X = {a,b,c}. The fuzzy sets λ, μ and γ are defined on X as follows:

λ:X → [0,1] defined as λ(a) = 0.9 ; λ(b) = 0.6; λ(c) = 0.8.

μ:X → [0,1] defined as μ(a) = 0.8 ; μ(b) = 0.7; μ(c) = 0.5.

γ:X → [0,1] defined as γ(a) = 0.7; γ(b) = 0.7; γ(c) = 0.5

Then T = {0, λ, μ, γ, (λ ʌ μ), (λ ʌ γ), (λ v μ),1} is fuzzy topology on X. Now cl(λ) = 1, cl(μ) = 1, cl(γ) = 1, cl(λ ʌμ) = 1, cl(λʌγ) = 1, cl(λvμ) = 1.

Therefore, every fuzzy open set in (X,T) is a fuzzy dense set.

Thus (X,T) is a fuzzy hyper-connected space.

Example 3.2

Let X = {a,b,c}. The fuzzy sets λ and μ are defined on X as follows:

λ:X → [0,1] defined as λ (a) = 0.1 ; λ(b) = 0.4; λ(c) = 0.9.

μ:X → [0,1] defined as μ(a) = 0.8 ; μ(b) = 0.7; μ(c) = 0.5.

Then T = {0, λ, μ, (λvμ), (λʌμ), 1} is fuzzy topology on X. Now cl(λ) = 1, cl(μ) = 1, cl(λ ʌ μ) = 1-(λ ʌ μ), cl(λ v μ) = 1.

In this example λ, μ, $(\lambda\vee\mu)$ are fuzzy dense but $(\lambda\wedge\mu)$ is not a fuzzy dense in (X,T).

Therefore, (X,T) is not a fuzzy hyper-connected space.

Theorem 3.1 [6]

If λ be a fuzzy open and fuzzy dense in a fuzzy Topology space (X,T) then $(1- \lambda)$ is a fuzzy nowhere dense set.

Proposition 3.1

If λ be a fuzzy open set in a fuzzy hyper-connected space then $1- \lambda$ is a fuzzy nowhere dense set.

Proof:

Let λ be a fuzzy open in a fuzzy hyper-connected space. Therefore λ is fuzzy dense in a fuzzy hyper connected space in (X,T).

Then by theorem 3.1, $(1-\lambda)$ is a fuzzy nowhere dense set.

Theorem 3.2 [6]

The complement of fuzzy open and fuzzy dense set in a fuzzy topological space is fuzzy nowhere dense set.

Proposition 3.2

Every fuzzy closed set in a fuzzy hyper-connected space is fuzzy nowhere dense set.

Proof:

Suppose λ_i be a fuzzy closed set in a fuzzy hyper-connected space.

We have every fuzzy open set in a fuzzy hyper-connected space is fuzzy dense.

Then by theorem 3.2,

Every closed set in a fuzzy hyper-connected space is fuzzy nowhere dense set.

Theorem 3.3 [6]

If λ be a fuzzy nowhere dense set in a fuzzy Topological space (X,T) then $1- \lambda$ is a fuzzy dense set in (X,T).

Proposition 3.3

The complement of fuzzy closed set in a fuzzy hyper-connected space is fuzzy dense.

Proof:

By preposition 3.2, every fuzzy closed set in a fuzzy hyper-connected space is fuzzy nowhere dense set.

By theorem 3.3, $1- \lambda$ is fuzzy dense in (X,T). Hence the complement of fuzzy closed set in a fuzzy hyper-connected space is a fuzzy dense set.

Proposition 3.4

Every fuzzy closed set in a fuzzy hyper-connected space is fuzzy nowhere dense in (X,T).

Proof:

Suppose that if the closed set λ is not a fuzzy nowhere dense set in a fuzzy hyper-connected space, therefore int $cl(\lambda) \neq 0$, and the complement of λ is fuzzy open but not a fuzzy dense in (X,T), this is a contradiction to the definition of fuzzy hyper-connected space. Hence every fuzzy closed set in fuzzy hyper-connected space is a fuzzy nowhere dense set in (X,T).

Proposition 3.5

A fuzzy hyper-connected space is a fuzzy second category space.

Proof:

In example 3.1, (X,T) is a fuzzy hyper-connected space, then $(1- \lambda)$, $(1- \mu)$, $(1-\gamma)$, $(1- \lambda\wedge\mu)$, $(1- \lambda\wedge\gamma)$, $(1- \lambda\vee\mu)$ are fuzzy nowhere dense sets. Now the union of fuzzy nowhere dense sets is not equal to 1. Therefore (X,T) is fuzzy second category space.

Theorem 3.4 [6]

A Second category space need not be fuzzy Baire space.

Proposition 3.6

A fuzzy hyper-connected space need not be fuzzy Baire space.

Proof:

Let (X,T) be a fuzzy hyper-connected space. By proposition 3.5, (X,T) is fuzzy second category space and by theorem 3.4, (X,T) need not be fuzzy baire space.

Proposition 3.7

A fuzzy hyper-connected space is a fuzzy D-Baire space.

Proof:

In example 3.1, (X,T) is a fuzzy hyper-connected space, then (1- λ), (1- μ), (1-γ), (1- $\lambda\wedge\mu$), (1- $\lambda\wedge\gamma$), (1- $\lambda\vee\mu$) are fuzzy nowhere dense sets. Now the union of fuzzy nowhere dense sets are fuzzy nowhere dense in (X,T). Therefore (X,T) is fuzzy D-Baire space.

Theorem 3.5 [5]

Every fuzzy D-Baire space is a fuzzy Baire space.

Proposition 3.8

A fuzzy hyper-connected space is a fuzzy Baire space.

Proof:

Let (X,T) be a fuzzy hyper-connected space. By proposition 3.7, (X,T) is a fuzzy D-Baire space and by theorem 3.5, (X,T) is a fuzzy Baire space.

There are some implications are hold.

Fuzzy hyper-connected space → Second category space

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Fuzzy D-Baire space

↓

Fuzzy Baire space

IV. REFERENCES

1. K.K.Azad., On fuzzy semi continuity, J.Math. Anal. Appl. 82 (1981) 14-32.
2. C.L.Chang, J.Math. Anal. Appl. 24, (1968), 182-190.
3. L.A Steen and J.A. Seebach, Jr., Counter examples in topology, Springer, New York, 1978.
4. G.Thangaraj and S.Anbazhagan, Some remarks on fuzzy P-Spaces, General Mathematics Notes 26 (1) (2015) 8 – 16.
5. SG.Thangaraj and S.Anjalmose, Fuzzy D-Baire Spaces, Annals of Fuzzy Mathematics and Informatics, 7(1) (2014) 99-108.
6. G.Thangaraj and S.Anjalmose, On Fuzzy Baire Spaces, Journal of Fuzzy Math., 21(3) (2013) 667-676.
7. G.Thangaraj and G.Balasubramanian, , J. Fuzzy Math. 11(3), (2003), 725-736.
8. L.A.Zadeh, Fuzzy Sets, Information and Control, 8 (1965) 338-353.